

ELIPSA

1. $S[0;0]$

$$L: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$K[0;6] \in N$$

$$K: \frac{0}{a^2} + \frac{36}{b^2} = 1$$

$$L[16;273] \in N$$

$$b^2 = 36$$

$$L: \frac{(76)^2}{a^2} + \frac{(273)^2}{b^2} = 1$$

$$\frac{6}{a^2} + \frac{12}{36} = 1$$

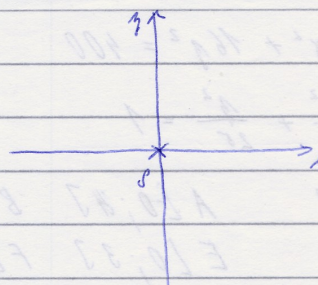
$$\frac{6}{a^2} + \frac{1}{3} = 1 \quad | \cdot 3a^2$$

$$18 + a^2 = 3a^2$$

$$2a^2 = 18$$

$$a^2 = 9$$

$$L: \frac{x^2}{9} + \frac{y^2}{36} = 1$$



2. $|EF| = 4 \quad N = 2$

$$A[-3;0]; B[3;0]$$

$$\rightarrow S = \frac{A+B}{2} = [0;0] \quad a = 3$$

$$\Leftrightarrow EF \parallel x$$

$$a^2 = b^2 + c^2$$

$$9 = b^2 + 4$$

$$b^2 = 5$$

$$L: \frac{x^2}{9} + \frac{y^2}{5} = 1$$

3. $E, F; a, b, N, A, B, C, D$

a) $L: 9x^2 + 25y^2 = 4 \quad | : 4$

$$\frac{x^2}{\frac{4}{9}} + \frac{y^2}{\frac{4}{25}} = 1$$

$$a = \frac{2}{3}$$

$$b = \frac{2}{5}$$

$$a > b$$

$$\Leftrightarrow EF \parallel N$$

$$a^2 = c^2 + b^2$$

$$\frac{4}{9} = c^2 + \frac{4}{25}$$

$$c^2 = \frac{4}{9} - \frac{4}{25} = \frac{64}{225}$$

$$c = \frac{8}{15}$$

$$S[0;0] \quad E[-\frac{8}{15}; 0]$$

$$F[\frac{8}{15}; 0]$$

$$A[-\frac{2}{3}; 0]$$

$$B[\frac{2}{3}; 0]$$

$$C[0; \frac{2}{5}]$$

$$D[0; -\frac{2}{5}]$$

$$b) \quad 25x^2 + 16y^2 = 400$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

$$\boxed{a=4} \quad \boxed{A=5}$$

$$a < A$$

$$\Leftrightarrow EF \parallel y$$

$$A^2 = a^2 + c^2$$

$$25 = 16 + c^2$$

$$c^2 = 9$$

$$\boxed{c=3}$$

$$S[0,0] \quad A[-4,0] \quad B[4,0]$$

$$E[0,3] \quad F[0,-3]$$

$$C[0,5] \quad D[0,-5]$$

$$c) \quad x^2 + 2y^2 = 32 \quad | : 32$$

$$\frac{x^2}{32} + \frac{y^2}{16} = 1$$

$$a^2 = 32$$

$$A^2 = 16$$

$$\boxed{a=4\sqrt{2}}$$

$$\boxed{A=4}$$

$$\Leftrightarrow EF \parallel x$$

$$a^2 = c^2 + A^2$$

$$32 = c^2 + 16$$

$$c^2 = 16$$

$$\boxed{c=4}$$

$$S[0,0]$$

$$A[-4\sqrt{2},0]$$

$$B[4\sqrt{2},0]$$

$$C[0,4]$$

$$D[0,-4]$$

$$E[-4,0]$$

$$F[4,0]$$

$$d) \quad 25(x-3)^2 + 9(y+2)^2 = 4 \quad | : 4$$

$$\frac{(x-3)^2}{\frac{4}{25}} + \frac{(y+2)^2}{\frac{4}{9}} = 1$$

$$S[3,-2]$$

$$\boxed{a=\frac{2}{5}}$$

$$\boxed{A=\frac{2}{3}}$$

$$A > a \Leftrightarrow EF \parallel y$$

$$A[\frac{13}{5}, -2]$$

$$B[\frac{17}{3}, -2]$$

$$C[3, -\frac{8}{3}]$$

$$D[3, -\frac{8}{3}]$$

$$E[3, -\frac{22}{15}]$$

$$F[3, -\frac{38}{15}]$$

$$A^2 = a^2 + c^2$$

$$c^2 = \frac{4}{9} - \frac{4}{25} = \frac{64}{225}$$

$$\boxed{c=\frac{8}{15}}$$

$$e) \quad 2x^2 + (y-4)^2 = 32 \quad | : 32$$

$$\frac{x^2}{16} + \frac{(y-4)^2}{32} = 1$$

$$\boxed{a=4}$$

$$A^2 = 32$$

$$\boxed{A=4\sqrt{2}}$$

$$a < A$$

$$\Leftrightarrow EF \parallel y$$

$$A^2 = c^2 + a^2$$

$$32 = c^2 + 16$$

$$c^2 = 16$$

$$\boxed{c=4}$$

$$S[0,4]$$

$$A[-4,4]$$

$$B[4,4]$$

$$C[0,4+4\sqrt{2}]$$

$$E[0,4]$$

$$D[0,4-4\sqrt{2}]$$

$$F[0,-4]$$

$$4. \Leftrightarrow EF \parallel X \quad \Leftrightarrow EF = X \quad S[mv; 0]$$

$$a) \quad a = 3 \\ k = 5$$

$$\frac{(x-mv)^2}{9} + \frac{(q-0)^2}{25} = 1 \quad | \cdot 225$$

$$\boxed{x: 25(x-mv)^2 + 9q^2 = 225}$$

$$b) \quad a = 10$$

$$a^2 = k^2 + l^2$$

$$100 = k^2 + 64$$

$$|EF| = 16 \Rightarrow l = 8$$

$$k^2 = 36$$

$$k = 6$$

$$\frac{(x-mv)^2}{100} + \frac{(q-0)^2}{36} = 1$$

$$36(x-mv)^2 + 100q^2 = 3600$$

$$\boxed{x: 9(x-mv)^2 + 25q^2 = 900}$$

$$5. \quad S[mv, mv] \quad \Leftrightarrow EF \parallel q \quad \Leftrightarrow EF = q \Rightarrow S[0; mv]$$

$$a) \quad a = 7 \\ k = 9$$

$$k^2 = a^2 + l^2$$

$$81 = 49 + l^2 \quad l^2 = 32$$

$$\frac{x^2}{49} + \frac{(q-mv)^2}{81} = 1$$

$$\boxed{81x^2 + 49(q-mv)^2 = 3969}$$

$$b) \quad |EF| = 6 \quad k = 8 \\ l = 3$$

$$k^2 = a^2 + l^2$$

$$64 = a^2 + 9$$

$$a^2 = 55$$

$$\frac{x^2}{55} + \frac{(q-mv)^2}{64} = 1$$

$$\boxed{64x^2 + 55(q-mv)^2 = 3520}$$

$$c) \quad |EF| = 6 \quad l = 0,6a \\ l = 3 \quad 3 = 0,6a$$

$$a = 5$$

$$k^2 = 9 + 25$$

$$k^2 = 34$$

$$\frac{x^2}{25} + \frac{(q-mv)^2}{34} = 1$$

$$\boxed{34x^2 + 25(q-mv)^2 = 850}$$

$$d) \quad a = 10$$

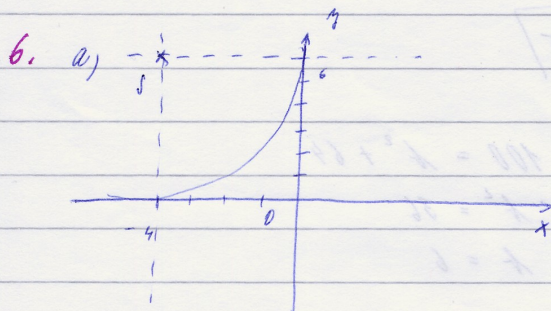
$$l = \frac{12}{13}a \quad l = \frac{120}{13}$$

$$k^2 = 100 + \frac{120^2}{169}$$

$$k^2 = \frac{31300}{169}$$

$$L: \frac{x^2}{100} + \frac{(y-m)^2}{\frac{31300}{169}} = 1$$

$$313x^2 + 169(y-m)^2 = 31300$$



$$S[-4, 6]$$

$$a = 4$$

$$L^2 = A^2 - a^2$$

$$A = 6$$

$$L^2 = 36 - 16$$

$$L^2 = 20$$

$$L: \frac{(x+4)^2}{16} + \frac{(y-6)^2}{36} = 1 \quad | \cdot 288$$

$$18(x+4)^2 + 8(y-6)^2 = 288 \quad | : 2$$

$$L: 9(x+4)^2 + 4(y-6)^2 = 144$$

1) $L[4, 0]$ $A[0, 5]$

$$S[4, 5] \quad a = 4$$

$$A = 5$$

$$\leftrightarrow EF \parallel A$$

$$L^2 = A^2 - a^2$$

$$L^2 = 25 - 16$$

$$L^2 = 9 \quad L = 3$$

$$L: \frac{(x-4)^2}{16} + \frac{(y-5)^2}{25} = 1$$

$$L: 25(x-4)^2 + 16(y-5)^2 = 400$$

7.

a) $25x^2 + 9y^2 - 100x - 54y - 44 = 0$

$$25[x^2 - 4x] + 9[y^2 - 6y] - 44 = 0$$

$$25[(x-2)^2 - 4] + 9[(y-3)^2 - 9] - 44 = 0$$

$$25(x-2)^2 + 9(y-3)^2 = 44 + 100 + 81$$

$$\frac{225}{225} \quad | : 225$$

$$L: \frac{(x-2)^2}{9} + \frac{(y-3)^2}{25} = 1$$

ANO

$$b) \quad 25x^2 + 7y^2 + 100x - 28y + 139 = 0$$

$$25[x^2 + 4x] + 7[y^2 - 4y] + 139 = 0$$

$$25[(x+2)^2 - 4] + 7[(y-2)^2 - 4] + 139 = 0$$

$$25(x+2)^2 + 7(y-2)^2 = \underbrace{-139 + 100 + 28}_{-11}$$

meni' romiceu

$$c) \quad 5x^2 + 7y^2 - 28y + 38 = 0$$

elipsa

$$5x^2 + 7[y^2 - 4y] + 38 = 0$$

$$5x^2 + 7[(y-2)^2 - 4] = -38$$

$$5x^2 + 7(y-2)^2 - 28 = -38$$

$$5x^2 + 7(y-2)^2 = \boxed{-10} \rightarrow \text{meni' romiceu elipsa}$$

$$d) \quad x^2 + 9y^2 - 4x + 9y = 0$$

$$(x-2)^2 - 4 + 9(y^2 + y) = 0$$

$$(x-2)^2 + 9\left[\left(y+\frac{1}{2}\right)^2 - \frac{1}{4}\right] = 4$$

$$(x-2)^2 + 9\left(y+\frac{1}{2}\right)^2 = 4 + \frac{9}{4}$$

$$(x-2)^2 + 9\left(y+\frac{1}{2}\right)^2 = \frac{25}{4} \quad | : \frac{25}{4}$$

$$\frac{4(x-2)^2}{25} + \frac{36\left(y+\frac{1}{2}\right)^2}{25} = 1$$

ANO

$$e) \quad 4x^2 + 9y^2 - 72x + 36y + 360 = 0$$

$$4[x^2 - 18x] + 9[y^2 + 4y] + 360 = 0$$

$$4[(x-9)^2 - 81] + 9[(y+2)^2 - 4] + 360 = 0$$

$$4(x-9)^2 - 324 + 9(y+2)^2 - 36 + 360 = 0$$

$$4(x-9)^2 + 9(y+2)^2 = \boxed{0} \rightarrow \text{meni' romiceu elipsa}$$

$$f) \quad x^2 - 9y^2 + 8x - 32 = 0$$

$$(x+4)^2 - 16 - 9y^2 = 32$$

$$(x+4)^2 - 9y^2 = 48$$

meni' romiceu elipsa (si si hiperbola)

$$8. \quad x: \quad 9x^2 + 4y^2 = 36 \rightarrow (3x)^2 + 4y^2 = 36$$

$$a) \quad p: \quad 3x + 2y - 6 = 0$$

$$3x = 2y - 6$$

$$(2y-6)^2 + 4y^2 = 36$$

$$4q^2 - 24q + 36 + 4q^2 = 36$$

$$8q^2 - 24q = 0$$

$$8(q^2 - 3q) = 0 \quad | : 8$$

$$3(q - 3) = 0$$

$$q_1 = 0$$

$$x_1 = -2$$

$$q_2 = 3$$

$$x_2 = 0$$

$$3x = 2q - 6$$

$$x = \frac{2q - 6}{3}$$

$$P_1 [-2; 0] ; P_2 [0; 3]$$

- přímka je nulová

$$h) p: x - q + 6 = 0$$

$$x = q - 6$$

$$9(q - 6)^2 + 4q^2 = 36$$

$$9(q^2 - 12q + 36) + 4q^2 = 36$$

$$9q^2 - 108q + 324 + 4q^2 = 36$$

$$13q^2 - 108q + 288 = 0$$

$$q_{1/2} = \frac{108 \pm \sqrt{3312}}{26}$$

nema' řešení

Přímka nema' s elipsou žádný společný bod. (aniž přímka).

$$c) p: 3\sqrt{3}x + 2q - 12 = 0$$

$$2q = 12 - 3\sqrt{3}x$$

$$e: 9x^2 + 4q^2 = 36$$

$$9x^2 + (2q)^2 = 36$$

$$9x^2 + (12 - 3\sqrt{3}x)^2 = 36$$

$$9x^2 + 144 - 72\sqrt{3}x + 27x^2 = 36$$

$$36x^2 - 72\sqrt{3}x + 108 = 0$$

$$x^2 - 2\sqrt{3}x + 3 = 0$$

$$x_{1/2} = \frac{2\sqrt{3} \pm \sqrt{0}}{2} = \sqrt{3} \quad x = \sqrt{3}$$

$$2q = 12 - 3\sqrt{3}\sqrt{3}$$

$$2q = 12 - 9$$

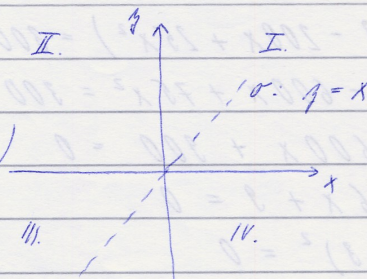
$$q = \frac{3}{2}$$

$$P \left[\sqrt{3}; \frac{3}{2} \right]$$

Přímka je tečnou elipsy.

9. $\ell: x^2 + 2y^2 = 18$

$r: y = x$ (ova I. a III. kvadrantu)



$\ell \cap r: x^2 + 2x^2 = 18$

$3x^2 = 18$

$x^2 = 6$

$x = \pm\sqrt{6} \quad y = \pm\sqrt{6}$

$P_1 [\sqrt{6}; \sqrt{6}]$

$P_2 [-\sqrt{6}; -\sqrt{6}]$

Adina $|P_1 P_2| = \sqrt{(-\sqrt{6}-\sqrt{6})^2 + (-\sqrt{6}-\sqrt{6})^2}$

$|P_1 P_2| = \sqrt{(-2\sqrt{6})^2 + (-2\sqrt{6})^2} = \sqrt{48} = 4\sqrt{3}$

10.

a) $p: 3x - 4y - 3 = 0$

$3x = 4y + 3$

$x = \frac{4}{3}y + 1$

$\ell: 3x^2 + 5y^2 = 120$

$\ell \cap p: 3\left(\frac{4}{3}y + 1\right)^2 + 5y^2 = 120$

$3\left(\frac{16}{9}y^2 + \frac{8}{3}y + 1\right) + 5y^2 = 120$

$\frac{16}{3}y^2 + 8y + 3 + 5y^2 = 120 \quad | \cdot 3$

$16y^2 + 24y + 9 + 15y^2 = 360$

$31y^2 + 24y - 351 = 0$

$y_{1,2} = \frac{-24 \pm \sqrt{210}}{62}$

$y_1 = 3$

$y_2 = -\frac{234}{62} = -\frac{117}{31}$

$x_1 = \frac{4}{3} \cdot 3 + 1 = 5$

$x_2 = \frac{4}{3} \left(-\frac{117}{31}\right) + 1 = -\frac{375}{93} = -\frac{125}{31}$

$P_1 [5; 3]$

$P_2 \left[-\frac{125}{31}; -\frac{117}{31}\right]$

b) $p: 5x + y - 20 = 0$

$y = 20 - 5x$

$\ell: 25x^2 + 3y^2 = 300$

$\ell \cap p: 25x^2 + 3(20 - 5x)^2 = 300$

$$25x^2 + 3(400 - 200x + 25x^2) = 300$$

$$25x^2 + 1200 - 600x + 75x^2 = 300$$

$$100x^2 - 600x + 900 = 0 \quad | :100$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$\underline{x = 3}$$

$$q = 20 - 5x$$

$$\underline{q = 20 - 15 = 5}$$

$$P[3; 5]$$

$$c) \quad p: x + 2q - 15 = 0$$

$$x: 3x^2 + 2q^2 = 84$$

$$x = -2q + 15$$

$$\text{L.p.} \quad 3(15 - 2q)^2 + 2q^2 = 84$$

$$3(225 - 60q + 4q^2) + 2q^2 = 84$$

$$675 - 180q + 12q^2 + 2q^2 = 84$$

$$14q^2 - 180q + 591 = 0$$

$$q_{1/2} = \frac{180 \pm \sqrt{(-180)^2 - 4 \cdot 14 \cdot 591}}{2 \cdot 14}$$

namaji' spoleiny' bod

$$d) \quad p = \{ [t+1; 2t-1]; t \in \mathbb{R} \}$$

$$x: 9x^2 + 16q^2 = 144$$

L.p.:

$$9(t+1)^2 + 16(2t-1)^2 = 144$$

$$9(t^2 + 2t + 1) + 16(4t^2 - 4t + 1) = 144$$

$$9t^2 + 18t + 9 + 64t^2 - 64t + 16 = 144$$

$$73t^2 - 46t - 119 = 0$$

$$t_{1/2} = \frac{46 \pm \sqrt{46^2 + 4 \cdot 73 \cdot 119}}{2 \cdot 73} = \frac{46 \pm 182}{146}$$

$$t_1 = \frac{238}{146} = \frac{119}{73}$$

$$P\left[\frac{119}{73}; \frac{165}{73}\right]$$

$$t_2 = -1$$

$$P[0; -3]$$

11. alebo povi 1 = jidom a mieu...

$p: q = x + q$

$L: 9x^2 + 16q^2 = 144$

$p \cap L: 9x^2 + 16(x+q)^2 = 144$

$9x^2 + 16(x^2 + 2xq + q^2) = 144$

$9x^2 + 16x^2 + 32xq + 16q^2 = 144$

$\underbrace{25x^2}_A + \underbrace{32xq}_B + \underbrace{16q^2 - 144}_C = 0$

$D \geq 0 \quad (32q)^2 - 4 \cdot 25(16q^2 - 144) \geq 0$

$1024q^2 - 100(16q^2 - 144) \geq 0$

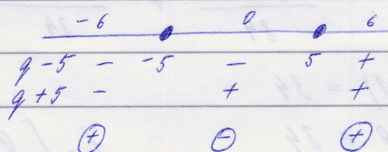
$-576q^2 + 14400 \geq 0$

$576q^2 \leq 14400 \quad | : 576$

$q^2 \leq 25$

$q^2 - 25 \leq 0$

$(q-5)(q+5) \leq 0$



$q \in (-5; 5)$

12. A [6; 3]

B [0; 1]

C [1; 3]

D [3; -2]

E [-2; 4]

F [0; 2.5]

$L: 3x^2 + 11q^2 = 41$

$L: 3x^2 + 11q^2 - 41 = 0$

A: $3 \cdot 36 + 11 \cdot 9 - 41 = 166 > 0$ VNE

B: $3 \cdot 0 + 11 \cdot 1 - 41 = -30 < 0$ VNITR

C: $3 \cdot 1 + 11 \cdot 9 - 41 = 61 > 0$ VNE

D: $3 \cdot 9 + 11 \cdot 4 - 41 = 30 > 0$ VNE

E: $3 \cdot 4 + 11 \cdot 16 - 41 = 147 > 0$ VNE

F: $3 \cdot 0 + 11 \cdot 6,25 - 41 = 27,75 > 0$ VNE

13. $L: 4x^2 + 9q^2 - 24x + 18q + 11 = 0$

max $x: q = 0$

$x \cap L:$

$4x^2 - 24x + 11 = 0$

$P = x \cap L \rightarrow P[x; 0]$

$x_{1/2} = \frac{24 \pm \sqrt{400}}{8} = \frac{24 \pm 20}{8}$

$$x_1 = \frac{44}{8} = \frac{11}{2} = 5,5$$

$$T_1 [5,5; 0]$$

$$x_2 = \frac{4}{8} = \frac{1}{2}$$

$$T_2 [0,5; 0]$$

$$u: 4[x^2 - 6x] + 9[q^2 + 2q] + 11 = 0$$

$$4(x-3)^2 - 36 + 9(q+1)^2 - 9 + 11 = 0$$

$$4(x-3)^2 + 9(q+1)^2 = 34 \quad | : 34$$

$$\frac{(x-3)^2}{\frac{34}{4}} + \frac{(q+1)^2}{\frac{34}{9}} = 1$$

$$t_1: T_1 [5,5; 0]$$

$$\frac{2,5(x-3)}{\frac{34}{4}} + \frac{(q+1)}{\frac{34}{9}} = 1$$

$$\frac{10(x-3)}{34} + \frac{9(q+1)}{34} = 1 \quad | \cdot 34$$

$$10(x-3) + 9(q+1) = 34$$

$$10x - 30 + 9q + 9 = 34$$

$$t_1: 10x + 9q - 55 = 0$$

$$t_2: [0,5; 0]$$

$$\frac{-2,5(x-3)}{\frac{34}{4}} + \frac{(q+1)}{\frac{34}{9}} = 1$$

$$-\frac{10(x-3)}{34} + \frac{9(q+1)}{34} = 1$$

$$-10(x-3) + 9(q+1) = 34$$

$$-10x + 30 + 9q + 9 = 34$$

$$t_2: 10x - 9q - 5 = 0$$

$$t_1 \cap \text{row } q: q: x=0$$

$$x: x^2 + 2q^2 - 8q = 0$$

$$x^2 + 2(q-2)^2 - 8 = 0$$

$$q \cap u: 0 + 2q^2 - 8q = 0 \quad | : 2$$

$$q^2 - 4q = 0$$

$$q(q-4) = 0$$

$$x^2 + 2(q-2)^2 = 8 \quad | : 8$$

$$\frac{x^2}{8} + \frac{(q-2)^2}{4} = 1$$

$$T_1 [0,0]$$

$$\frac{0 \cdot x}{8} + \frac{(0-2)(q-2)}{4} = 1 \quad | \cdot 4$$

$$-2q + 4 = 4$$

$$q = 0$$

$$q_1 = 0 \quad q_2 = 4$$

$$T_2 [0; 4]$$

$$\frac{0 \cdot x}{8} + \frac{(4-2)(q-2)}{4} = 1$$

$$2q - 4 = 4$$

$$\underline{q = 4}$$

$$14. \quad x: 3x^2 + 6q^2 = 18$$

$$\perp p: x - q = 0 \quad \bar{m}_p = (1; -1)$$

$$\bar{m}_l = (1; 1)$$

$$t: x + q + c = 0$$

$$x = -q - c$$

$$t \cap x: 3(-q-c)^2 + 6q^2 = 18$$

$$3(q^2 + 2qc + c^2) + 6q^2 = 18$$

$$3q^2 + 6qc + 3c^2 + 6q^2 = 18$$

$$9q^2 + 6qc + 3c^2 - 18 = 0$$

$$A = 9 \quad B = 6c \quad C = 3c^2 - 18$$

$$t - \text{normal}$$

$$D = 0$$

$$D = 36c^2 - 4 \cdot 9(3c^2 - 18)$$

$$D = 36c^2 - 108c^2 + 648 = 0$$

$$72c^2 = 648$$

$$c^2 = 9$$

$$c = \pm 3$$

$$t_1: x + q + 3 = 0$$

$$t_2: x + q - 3 = 0$$

$$15. \quad x: 9x^2 + 16q^2 = 144$$

$$t: q = x + c \quad k = 1$$

$$x \cap t: 9x^2 + 16(x+c)^2 = 144$$

$$9x^2 + 16(x^2 + 2xc + c^2) = 144$$

$$25x^2 + 32xc + 16c^2 - 144 = 0$$

$$A = 25 \quad B = 32c \quad C = 16c^2 - 144$$

$$D = 0$$

$$(32c)^2 - 4 \cdot 25(16c^2 - 144) = 0$$

$$1024c^2 - 1600c^2 + 14400 = 0$$

$$576c^2 = 14400$$

$$c^2 = 25$$

$$c = \pm 5$$

$$t_1: x - q + 5 = 0$$

$$t_2: x - q - 5 = 0$$

$$16. \quad \ell: 9x^2 + 16y^2 = 144$$

$$p = t: x - y + a = 0$$

$$x = y - a$$

$$\ell \cap t: 9(y-a)^2 + 16y^2 = 144$$

$$9(y^2 - 2ay + a^2) + 16y^2 = 144$$

$$25y^2 - 18ay + 9a^2 - 144 = 0$$

$$A = 25 \quad B = -18a \quad C = 9a^2 - 144$$

$$D = 0$$

$$(-18a)^2 - 4 \cdot 25(9a^2 - 144) = 0$$

$$324a^2 - 900a^2 + 14400 = 0$$

$$576a^2 = 14400$$

$$a^2 = 25$$

$$a = \pm 5$$

$$t_1: x - y + 5 = 0$$

$$t_2: x - y - 5 = 0$$

$$17. a) \quad \ell: 25x^2 + a^2y^2 = 144$$

$$p: 2x + 3y - 12 = 0$$

$$2x = -3y + 12$$

$$x = -\frac{3}{2}y + 6$$

$$\ell \cap p: 25\left(6 - \frac{3}{2}y\right)^2 + a^2y^2 = 144$$

$$25\left(36 - 18y + \frac{9}{4}y^2\right) + a^2y^2 = 144$$

$$900 - 450y + \frac{225}{4}y^2 + a^2y^2 = 144 \quad | \cdot 4$$

$$3600 - 1800y + 225y^2 + 4a^2y^2 = 576$$

$$y^2 \underbrace{(225 + 4a^2)}_A + y \underbrace{(-1800)}_B + \underbrace{3024}_C = 0$$

$$D = 0$$

$$(-1800)^2 - 4(225 + 4a^2) \cdot 3024 = 0$$

$$3240000 - 2721600 + 48384a^2 = 0$$

$$48384a^2 = -518400$$

NELZE

no existing

$$b) \quad \ell: 25x^2 + a^2y^2 = 25a^2$$

$$p: 2x + 3y - 12 = 0$$

$$a \neq 0$$

$$x = -\frac{3}{2}y + 6$$

$$25\left(-\frac{3}{2}y + 6\right)^2 + a^2y^2 = 25a^2$$

$$25\left(\frac{9}{4}y^2 - 18y + 36\right) + a^2y^2 = 25a^2$$

$$\frac{225}{4}q^2 - 450q + 900 + a^2q^2 = 25a^2 \quad (1 \cdot 4) \text{ dan } (x^2 - 2x) \text{ dan } : 4$$

$$225q^2 - 1800q + 3600 + 4a^2q^2 = 100a^2 \quad \text{dan } (x^2 - 2x) \text{ dan } : 4$$

$$225q^2 - 1800q + 3600 + 4a^2q^2 - 100a^2 = 0 \quad (x^2 - 2x) \text{ dan } : 4$$

$$q^2 \underbrace{(225 + 4a^2)}_A - \underbrace{1800q}_B + \underbrace{3600 - 100a^2}_C = 0 \quad (x^2 - 2x) \text{ dan } : 4$$

$$D = 0 \quad (-1800)^2 - 4 \cdot (225 + 4a^2) \cdot (3600 - 100a^2) = 0$$

$$18^2 \cdot 100^2 - 4 \cdot 100 (225 + 4a^2) (36 - a^2) = 0$$

$$100 [18^2 \cdot 100 - 4 (225 + 4a^2) (36 - a^2)] = 0 \quad : 100$$

$$32400 - (900 + 16a^2)(36 - a^2) = 0$$

$$32400 - (32400 - 900a^2 + 576a^2 - 16a^4) = 0$$

$$16a^4 + 324a^2 = 0$$

$$4a^2(4a^2 + 81) = 0$$

$$a^2 = 0$$

$$a = 0$$

$$\text{atau } a \neq 0$$

$$4a^2 = -81$$

tidak

tidak ada solusi nyata.

18.

$$a) \quad x: 49x^2 + 100q^2 - 294x + 400q - 4059 = 0 \quad T[9; q]$$

$$T: 49 \cdot 81 + 100q^2 - 294 \cdot 9 + 400q - 4059 = 0$$

$$3969 + 100q^2 - 2646 + 400q - 4059 = 0$$

$$100q^2 + 400q - 2736 = 0$$

$$q_{1/2} = \frac{-400 \pm 1120}{200} = \frac{-40 \pm 112}{20}$$

$$q_{1/2} = \begin{cases} 3,6 \\ -7,6 \end{cases}$$

$$T_1[9; 3,6]$$

$$T_2[9; -7,6]$$

$$e: 49(x^2 - 6x) + 100(q^2 + 4q) - 4059 = 0$$

$$49(x-3)^2 - 49 \cdot 9 + 100(q+2)^2 - 100 \cdot 4 = 4059$$

$$49(x-3)^2 + 100(q+2)^2 = 4900$$

$$\frac{(x-3)^2}{100} + \frac{(q+2)^2}{49} = 1$$

$$T_1 [9; 3, 6] \quad \frac{(9-3)(x-3)}{100} + \frac{(3,6+2)(q+2)}{49} = 1$$

$$49 \cdot 6 \cdot (x-3) + 100 \cdot 5,6 \cdot (q+2) = 4900$$

$$294(x-3) + 560(q+2) = 4900$$

$$294x - 882 + 560q + 1120 = 4900$$

$$294x + 560q + 238 = 4900$$

$$t_1: 294x + 560q - 4662 = 0 \quad 147x + 280q - 2331 = 0$$

$$\boxed{21x + 40q - 333 = 0}$$

$$T_2 [9; -7, 6] \quad \frac{6(x-3)}{100} + \frac{-5,6(q+2)}{49} = 1 \quad | \cdot 4900$$

$$294(x-3) - 560(q+2) = 4900$$

$$294x - 882 - 560q - 1120 = 4900$$

$$294x - 560q - 6802 = 0$$

$$t_2: 147x - 280q - 3401 = 0 \quad | : 7$$

$$\boxed{21x - 40q - 483 = 0}$$

$$h) \quad e: 9x^2 + 16q^2 - 54x + 64q - 129 = 0 \quad T[8; 3]$$

$$T: 9 \cdot 64 + 16q^2 - 432 + 64q - 129 = 0$$

$$16q^2 + 64q + 15 = 0$$

$$q_{1/2} = \frac{-64 \pm 56}{32} =$$

$$-\frac{1}{4} = -0,25$$

$$-3,75$$

$$T_1 [8; -0,25] \quad T_2 [8; -3,75]$$

$$e: 9(x^2 - 6x) + 16(q^2 + 4q) - 129 = 0$$

$$9(x-3)^2 - 81 + 16(q+2)^2 - 64 - 129 = 0$$

$$9(x-3)^2 + 16(y+2)^2 = 274$$

$$T_1 [8; -0,25]$$

$$9 \cdot (8-3)(x-3) + 16 \left(-\frac{1}{4} + 2\right)(y+2) = 274$$

$$45(x-3) + 28(y+2) = 274$$

$$t_1: 45x + 28y - 353 = 0$$

$$T_2 [8; -3,75]$$

$$9(8-3)(x-3) + 16(-3,75+2)(y+2) = 274$$

$$45(x-3) - 28(y+2) = 274$$

$$45x - 135 - 28y - 56 = 274$$

$$45x - 28y - 465 = 0$$

19.

$$a) \quad c: 5x^2 + 9y^2 = 45$$

$$M [0; -3]$$

$$t: y = kx + q$$

$$M \in t: -3 = 0 \cdot x + q \Rightarrow q = -3$$

$$y = kx - 3$$

$$t \cap c:$$

$$5x^2 + 9(kx-3)^2 = 45$$

$$5x^2 + 9(k^2x^2 - 6kx + 9) = 45$$

$$5x^2 + 9k^2x^2 - 54kx + 81 = 45$$

$$x^2(5+9k^2) - 54kx + 36 = 0$$

$$D = 0$$

$$(-54k)^2 - 4 \cdot 36(5+9k^2) = 0$$

$$2916k^2 - 720 - 1296k^2 = 0$$

$$1620k^2 = 720$$

$$k^2 = \frac{72}{162} = \frac{36}{81} \quad k = \pm \frac{6}{9} = \pm \frac{2}{3}$$

$$t_1: y = \frac{2}{3}x - 3 \quad / \cdot 3$$

$$2x - 3y - 9 = 0$$

$$M_{t_1} = (-2; -3)$$

$$\cos \varphi = \frac{14-91}{113 \cdot 1193}$$

$$t_2: y = -\frac{2}{3}x - 3 \quad / \cdot 3$$

$$2x + 3y + 9 = 0$$

$$M_{t_2} = (2; -3)$$

$$\varphi = 67^\circ 23'$$

$$h) \quad x: 9x^2 + 4y^2 = 36 \quad M[5; 0] \quad 155 = (2+p)M + (8-x)C$$

$$t: \quad y = kx + q \quad (2+p)(2+\frac{1}{2})M + (8-x)(8-y) \cdot C \quad [22, 0; 10] T$$

$$M \in t: \quad 0 = 5k + q \quad q = -5k \quad q = kx - 5k$$

$$\text{in } t: \quad 9x^2 + 4(kx - 5k)^2 = 36$$

$$9x^2 + 4(k^2x^2 - 10k^2x + 25k^2) = 36 \quad (C-B)C$$

$$9x^2 + 4k^2x^2 - 40k^2x + 100k^2 = 36$$

$$x^2(9 + 4k^2) - 40k^2x + 100k^2 - 36 = 0$$

$$D = 0 \quad (-40k^2)^2 - 4(9 + 4k^2)(100k^2 - 36) = 0$$

$$1600k^4 - (36 + 16k^2)(100k^2 - 36) = 0$$

$$1600k^4 - 3600k^2 + 1296 - 1600k^4 - 576 = 0$$

$$3600k^2 = 720$$

$$k^2 = \frac{72}{360} = \frac{2}{10} = \frac{1}{5}$$

$$k = \pm \frac{1}{\sqrt{5}} = \pm \frac{\sqrt{5}}{5}$$

$$t_1: \quad y = \frac{\sqrt{5}}{5}x - \sqrt{5} \quad | \cdot 5$$

$$5x - 5y - 5\sqrt{5} = 0 \quad \bar{M}_{t_1} = (\sqrt{5}; -5)$$

$$t_2: \quad y = -\frac{\sqrt{5}}{5}x + \sqrt{5} \quad | \cdot 5 \quad \bar{M}_{t_2} = (\sqrt{5}; 5)$$

$$5x + 5y - 5\sqrt{5} = 0$$

$$\cos \varphi = \frac{15 - 251}{130 \cdot 130} = \frac{20}{30} = \frac{2}{3} \quad \varphi = 48^\circ 11'$$

$$c) \quad 9(x-2)^2 + 16(y+3)^2 = 144 \quad M[0; -3]$$

$$t: \quad y = kx + q$$

$$-3 = 0k + q \quad q = -3$$

$$t: \quad y = kx - 3$$

$$\text{in } t: \quad 9(x-2)^2 + 16(kx-3+3)^2 = 144$$

$$9(x^2 - 4x + 4) + 16k^2x^2 = 144$$

$$9x^2 - 36x + 36 + 16k^2x^2 = 144$$

$$x^2(9 + 16k^2) - 36x - 108 = 0$$

$$D = 0$$

$$(-36)^2 - 4(9 + 16k^2)(-108) = 0$$

$$1296 + 432(9 + 16k^2) = 0$$

$$1296 + 3888 + 6912k^2 = 0$$

$$6912k^2 = -5184$$

lingkaran tidak ada.

$$d) * \quad 3(x+2)^2 + 6(q-5)^2 = 216$$

$$M[3; -3]$$

$$t: \quad q = kx + q$$

$$-6 = 6k + q$$

$$q = -6 - 6k$$

$$t: \quad q = kx - 6 - 6k$$

$$t \wedge x: \quad 3(x+2)^2 + 6(kx - 6 - 6k - 5)^2 = 72$$

$$3(x^2 + 4x + 4) + 6(kx - 6k - 11)^2 = 72$$

$$3x^2 + 12x + 12 + 6(k^2x^2 + 36k^2 + 121 - 12k^2x - 22kx + 132k) = 72$$

$$3x^2 + 12x + 12 + 6k^2x^2 + 216k^2 + 726 - 72k^2x - 132kx + 792k = 72$$

$$3x^2(1 + 2k^2) + 3x(4 - 24k^2 - 44k) + 3(72k^2 + 264k + 222) = 0$$

/: 3

$$x^2 \underbrace{(1 + 2k^2)}_A + x \underbrace{(4 - 24k^2 - 44k)}_B + \underbrace{(72k^2 + 264k + 222)}_C = 0$$

$$D = 0$$

$$(4 - 24k^2 - 44k)^2 - 4 \cdot (1 + 2k^2)(72k^2 + 264k + 222) = 0$$

$$16 + 576k^4 + 1936k^2 - (4 + 8k^2)(72k^2 + 264k + 222) = 0$$

$$- 192k^2 - 352k + 2112k^3$$

$$576k^4 + 1744k^2 - 352k + 2112k^3 + 16 - 288k^2 - 1056k - 888$$

$$- 576k^4 - 2112k^3 - 1776k^2 = 0$$

$$1: 3(x+2)^2 + 6(q-5)^2 = 216 / :3 \quad M[2] = 37$$

$$2: (x+2)^2 + 2(q-5)^2 = 72$$

$$\text{not: } 0 = (301 - 1)(2301 + 2) + -3 = 3k + q, \quad q = -3 - 3k$$

$$x^2 + 4x + 4 + 2(kx - 3 - 3k)^2 = 72$$

$$x^2 + 4x + 4 + 2(k^2x^2 + 6k + 9k^2 - 16kx - 6k^2x + 48k) = 72$$

$$x^2(1+2k^2) + x(4-32k-12k^2) + 18k^2 + 96k + 60 = 0$$

$$D = 0$$

$$(4-32k-12k^2)^2 - 4(1+2k^2)(18k^2+96k+60) = 0$$

$$(4(1-8k-3k^2))^2 - 4(1+2k^2)(18k^2+96k+60) = 0$$

$$4^2(1-8k-3k^2)^2 - 4(1+2k^2)(18k^2+96k+60) = 0 \quad / :4$$

$$4(1+64k^2+9k^4-16k-6k^2+48k^3) - (18k^2+96k+60+36k^4)$$

$$4+256k^2+36k^4-64k-24k^2+192k^3-18k^2-96k-60-36k^4-192k^3-120k^2=0$$

$$94k^2-160k-56=0 \quad / :2$$

$$47k^2-80k-28=0$$

$$k_{1,2} = \frac{80 \pm 108}{94} = \begin{matrix} 2 \\ -\frac{28}{94} = -\frac{14}{47} \end{matrix}$$

$$t_1: q = 2x - 9$$

$$q_1 = -3 - 3 \cdot 2 = -9$$

$$4: 2x - q - 9 = 0$$

$$t_2: q = -\frac{14}{47}x - \frac{99}{47} \quad / \cdot 47$$

$$q_2 = -3 - 3 \cdot \left(-\frac{14}{47}\right) = -\frac{99}{47}$$

$$6: 14x + 47q + 99 = 0$$

$$m_{t_1} = (2, -1)$$

$$m_{t_2} = (14, 47)$$

$$\cos \varphi = \frac{12 \cdot 14 - 1 \cdot 47}{\sqrt{5} \sqrt{14^2 + 47^2}} = \frac{13}{15 \sqrt{12405}} = 0,173265086$$

$$\varphi = 80^\circ 01'$$

$$20. \quad x: \quad x^2 + 4q^2 - 6x - 16q - 11 = 0$$

$$(x-3)^2 - 9 + 4(q^2 - 4q) - 11 = 0$$

$$(x-3)^2 + 4(q-2)^2 - 16 - 9 - 11 = 0$$

$$(x-3)^2 + 4(q-2)^2 = 36 \quad | : 36$$

$$\frac{(x-3)^2}{36} + \frac{(q-2)^2}{9} = 1$$

$$a = 6$$

$$b = 3$$

$$\leftrightarrow EF \parallel x$$

$$S[3; 2]$$

$$C[3; 5]$$

$$A[-3; 2]$$

$$D[3; -1]$$

$$B[9; 2]$$

$$a^2 = b^2 + c^2$$

$$36 = 9 + c^2 \quad c^2 = 27$$

$$E[3-3\sqrt{3}; 2]$$

$$c = 3\sqrt{3}$$

$$F[3+3\sqrt{3}; 2]$$

$$T[x; 5]:$$

$$x^2 + 4 \cdot 25 - 6x - 16 \cdot 5 - 11 = 0$$

$$x^2 + 100 - 6x - 80 - 11 = 0$$

$$x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0$$

$$x-3 = 0$$

$$x = 3$$

$$T[3; 5]$$

$$t: (3-3)(x-3) + 4(5-2)(q-2) = 36$$

$$12(q-2) = 36$$

$$q-2 = 3$$

$$\underline{q = 5}$$

$$21. \quad 9x^2 + 16q^2 - 54x + 64q + 1 = 0$$

$$9(x^2 - 6x) + 16(q^2 + 4q) + 1 = 0$$

$$9(x-3)^2 - 81 + 16(q+2)^2 - 64 + 1 = 0$$

$$9(x-3)^2 + 16(q+2)^2 = 144 \quad | : 144$$

$$\frac{(x-3)^2}{16} + \frac{(q+2)^2}{9} = 1 \quad S[3; -2]$$

$$a = 4$$

$$\leftrightarrow EF \parallel x$$

$$t = 3$$

$$a^2 = t^2 + c^2$$

$$16 = 9 + c^2$$

$$c^2 = 7$$

$$c = \sqrt{7}$$

$$E[3 - \sqrt{7}; -2]$$

$$F[3 + \sqrt{7}; -2]$$

$$22. \quad S[0; 0]$$

$$A[8; 3] \quad B[6; 4]$$

$$c: \quad \frac{x^2}{a^2} + \frac{y^2}{t^2} = 1$$

$$A: \quad \frac{64}{a^2} + \frac{9}{t^2} = 1 \quad | \cdot a^2 t^2$$

$$64t^2 + 9a^2 = a^2 t^2$$

$$B: \quad \frac{36}{a^2} + \frac{16}{t^2} = 1 \quad | \cdot a^2 t^2$$

$$36t^2 + 16a^2 = a^2 t^2$$

$$64t^2 + 9a^2 = a^2 t^2 \quad | (-16)$$

$$36t^2 + 16a^2 = a^2 t^2 \quad | (-9t^2)$$

$$\begin{cases} -1024t^2 - 144a^2 = -16a^2 t^2 \\ 324t^2 + 144a^2 = -9a^2 t^2 \end{cases} \quad \oplus$$

$$-700t^2 = -25a^2 t^2 \quad | : t^2$$

$$-700 = -25a^2$$

$$a^2 = 28$$

$$36t^2 + 16 \cdot 28 = 28t^2$$

$$8t^2 = -448$$

$$t^2 = -56 \quad \text{nelik}$$

Pakora' elipsa neexistuje

$$23. \quad c: \quad 2x^2 + y^2 + 4x - 2y + 2 = 0$$

$$p: \quad y = kx + 1$$

$$c \cap p: \quad 2x^2 + (kx + 1)^2 + 4x - 2(kx + 1) + 2 = 0$$

$$2x^2 + k^2 x^2 + 2kx + 1 + 4x - 2kx - 2 + 2 = 0$$

$$2x^2 + k^2x^2 + 4x + 1 = 0 = 2x^2 + 12x + 1 = 0$$

$$x^2(2+k^2) + 4x + 1 = 0$$

$$D = 0 \quad 16 - 4(2+k^2) \cdot 1 = 0$$

$$4(2+k^2) = 16$$

$$2+k^2 = 4$$

$$k^2 = 2$$

$$k = \pm \sqrt{2}$$

... TEČNA

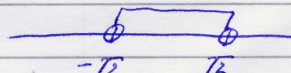
$$D > 0 \quad 16 - 4(2+k^2) > 0$$

$$16 - 8 - 4k^2 > 0$$

$$x = 93 \Rightarrow 8 - 4k^2 > 0$$

$$8 > 4k^2$$

$$k^2 < 2$$



$$k \in (-\sqrt{2}; \sqrt{2})$$

... SEČNA

$$D < 0 \quad 16 - 4(2+k^2) < 0$$

$$k^2 > 2$$

$$k \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$$

... VNEJŠÍ PŘÍMKA

24.

$$\Leftrightarrow EF \parallel x$$

$$\Leftrightarrow EF \equiv x$$

$$\Rightarrow x \in S[0; 0] \quad ; \quad k = 6$$

$$M[-4; \sqrt{21}]$$

$$L: \quad \frac{x^2}{a^2} + \frac{y^2}{k^2} = 1$$

$$M \in L: \quad \frac{16}{a^2} + \frac{21}{k^2} = 1$$

$$a^2 = k^2 + k^2$$

$$a^2 = k^2 + 36$$

$$\frac{16}{k^2 + 36} + \frac{21}{k^2} = 1 \quad | \cdot k^2(k^2 + 36)$$

$$16k^2 + 21(k^2 + 36) = k^2(k^2 + 36)$$

$$16k^2 + 21k^2 + 756 = k^4 + 36k^2$$

$$k^4 - 15k^2 - 756 = 0$$

$$q = t^2$$

$$q^2 - 15q - 756 = 0 = 1 + x + x^2 + x^3 + x^4$$

$$q_{1,2} = \frac{15 \pm \sqrt{9249}}{2} = \frac{15 \pm 57}{2}$$

$$q_1 = 36 = t^2$$

$$q_2 = -21 = t^2$$

$$a^2 = t^2 + 36$$

$$a^2 = 36 + 36$$

$$a^2 = 72$$

$$u: \frac{x^2}{72} + \frac{y^2}{36} = 1$$

$$25. \quad S[0;0] \quad w=5$$

$$t=3$$

$$\leftrightarrow EF \parallel x \quad \leftrightarrow EF \equiv x$$

$$F[4,0]$$

$$t$$

$$a^2 = t^2 + c^2$$

$$25 = 9 + c^2$$

$$c^2 = 16$$

$$c = 4$$

$$F[4;0]$$

$$E[4;0]$$

$$F[4;0]$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$p \perp x \Rightarrow p \parallel y \quad [0;0] \quad q: x=0$$

$$p: x+c=0$$

$$F \in p: 4+c=0$$

$$c = -4$$

$$x-4=0$$

$$p \cap u = p \quad x-4=0 \quad x=4$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \quad \frac{16}{25} + \frac{y^2}{9} = 1 \quad | \cdot 225$$

$$(25+y^2)/9 = (25+y^2)/16 \cdot 9 + 25y^2 = 225$$

$$25+y^2 = 225 + 16y^2 + 25y^2$$

$$0 = 225 + 16y^2 - 25y^2$$

$$y = \pm \frac{9}{5}$$

$$P_1 \left[4; \frac{9}{5} \right]$$

$$L: \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$P_2 \left[4; -\frac{9}{5} \right]$$

$$t_1: \frac{4x}{25} + \frac{\frac{9}{5}y}{9} = 1$$

$$t_2: \frac{4x}{25} - \frac{\frac{9}{5}y}{9} = 1$$

$$\frac{4}{25}x + \frac{1}{5}y = 1 \quad | \cdot 25$$

$$\frac{4}{25}x - \frac{1}{5}y = 1 \quad | \cdot 25$$

$$t_1: 4x + 5y - 25 = 0$$

$$t_2: 4x - 5y - 25 = 0$$

$$\bar{M}_{t_1} = (4; 5)$$

$$\bar{M}_{t_2} = (4; -5)$$

$$\cos \varphi = \frac{|1 \cdot 16 - 25|}{\sqrt{49} \sqrt{41}} = \frac{9}{41}$$

$$\varphi = 77^\circ 19'$$

26.

$$A[4; 0]$$

$$L: 3x^2 + 6y^2 = 18$$

$$t: y = kx + q$$

$$A \in t: 0 = 4k + q \quad q = -4k$$

$$y = kx - 4k$$

$$L \cap t: 3x^2 + 6(kx - 4k)^2 = 18$$

$$3x^2 + 6(k^2x^2 - 8k^2x + 16k^2) = 18 \quad | : 3$$

$$x^2 + 2(k^2x^2 - 8k^2x + 16k^2) = 6$$

$$x^2 + 2k^2x^2 - 16k^2x + 32k^2 = 6$$

$$x^2(1 + 2k^2) - 16k^2x + 32k^2 = 6$$

$$x^2 \underbrace{(1 + 2k^2)}_A - \underbrace{16k^2x}_B + \underbrace{32k^2 - 6}_C = 0$$

$$D = 0 \Rightarrow 256k^4 - 4(1 + 2k^2)(32k^2 - 6) = 0 \quad | : 4$$

$$64k^4 - (1 + 2k^2)(32k^2 - 6) = 0$$

$$64k^4 - 32k^2 + 6 - 64k^2 + 12k^2 = 0$$

$$-20k^2 - 6 = 0$$

$$20k^2 = 6 \Rightarrow k^2 = \frac{6}{20} = \frac{3}{10}$$

$$k = \pm \sqrt{\frac{3}{10}}$$

$$t_1: q = \sqrt{\frac{3}{10}}x - 4\sqrt{\frac{3}{10}} \quad | \cdot \sqrt{10}$$

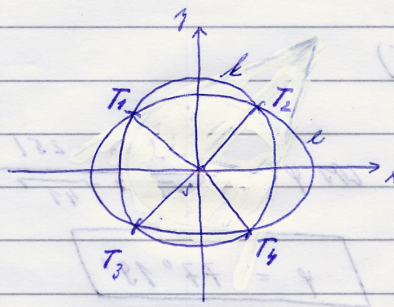
$$t_2: q = -\sqrt{\frac{3}{10}}x + 4\sqrt{\frac{3}{10}} \quad | \cdot \sqrt{10}$$

$$\boxed{\sqrt{3}x - \sqrt{10}q - 4\sqrt{3} = 0}$$

$$\boxed{\sqrt{3}x + \sqrt{10}q - 4\sqrt{3} = 0}$$

$$27. \quad c: 3x^2 + 9q^2 = 45 \quad S[0;0] \quad |St| = 3 = |ST|$$

Bodů dotyku dvou elipsů leží
na kružnici se středem S a polo-
mírem $r = 3$



$$k: x^2 + q^2 = 9$$

$$k \cap c = T$$

$$\left. \begin{array}{l} 3x^2 + 9q^2 = 45 \\ x^2 + q^2 = 9 \end{array} \right\} \begin{array}{l} |(-3) \\ \hline \end{array} \quad \oplus$$

$$q^2 = 18$$

$$q^2 = 3$$

$$q = \pm \sqrt{3}$$

$$x^2 = 6 \quad x = \pm \sqrt{6}$$

$$T_1 [\sqrt{6}; \sqrt{3}]$$

$$t_1: 3\sqrt{6}x + 9\sqrt{3}q - 45 = 0$$

$$T_2 [\sqrt{6}; -\sqrt{3}]$$

$$t_2: 3\sqrt{6}x - 9\sqrt{3}q - 45 = 0$$

$$T_3 [-\sqrt{6}; \sqrt{3}]$$

$$t_3: -3\sqrt{6}x + 9\sqrt{3}q - 45 = 0$$

$$T_4 [-\sqrt{6}; -\sqrt{3}]$$

$$t_4: 3\sqrt{6}x + 9\sqrt{3}q + 45 = 0$$

28.

$$a) \quad 9x^2 + 4q^2 - 36x - 8q + 4 = 0$$

$$S[2;1]$$

$$9[x^2 - 4x] + 4[q^2 - 2q] + 4 = 0$$

$$a = 2 \quad \leftrightarrow EF \parallel q$$

$$9(x-2)^2 - 36 + 4(q-1)^2 - 4 + 4 = 0$$

$$k = 3 \quad k > a$$

$$k^2 = a^2 + c^2$$

$$9(x-2)^2 + 4(q-1)^2 = 36 \quad | : 36$$

$$c^2 = 9 - 4 = 5 \quad c = \sqrt{5}$$

$$\frac{(x-2)^2}{4} + \frac{(q-1)^2}{9} = 1$$

$$E[2; 1+\sqrt{5}] \quad F[2; 1-\sqrt{5}]$$

$$\begin{aligned}
 4) \quad & 9x^2 + 4y^2 - 18x - 16y + 24 = 0 \\
 & 9[x^2 - 2x] + 4[y^2 - 4y] + 24 = 0 \\
 & 9(x-1)^2 - 9 + 4(y-2)^2 - 16 + 24 = 0 \\
 & 9(x-1)^2 + 4(y-2)^2 = 1 \\
 & \frac{(x-1)^2}{\frac{1}{9}} + \frac{(y-2)^2}{\frac{1}{4}} = 1
 \end{aligned}$$

$$S[1; 2]$$

$$a = \frac{1}{3} \leftrightarrow EF \parallel y$$

$$b = \frac{1}{2} \quad b > a$$

$$c^2 = b^2 - a^2$$

$$c^2 = \frac{1}{4} - \frac{1}{9} = \frac{9-4}{36} = \frac{5}{36}$$

$$c = \frac{\sqrt{5}}{6}$$

$$E[1; 2 + \frac{\sqrt{5}}{6}] \quad F[1; 2 - \frac{\sqrt{5}}{6}]$$

$$\begin{aligned}
 5) \quad & 3x^2 + 2y^2 + 6x - 12y + 21 = 0 \\
 & 3(x^2 + 2x) + 2(y^2 - 6y) + 21 = 0 \\
 & 3(x+1)^2 - 3 + 2(y-3)^2 - 18 + 21 = 0 \\
 & 3(x+1)^2 + 2(y-3)^2 = 0
 \end{aligned}$$

NENÍ ELIPSA

$$\begin{aligned}
 6) \quad & 6x^2 + 3y^2 - 12x + 13 = 0 \\
 & 6(x^2 - 2x) + 3y^2 + 13 = 0 \\
 & 6(x-1)^2 - 6 + 3y^2 + 13 = 0 \\
 & 6(x-1)^2 + 3y^2 = -7
 \end{aligned}$$

NENÍ ELIPSA

29. $k \in \mathbb{R}$

$$\begin{aligned}
 a) \quad & x^2 + 4y^2 - 4x - 8y + k = 0 \\
 & (x-2)^2 - 4 + 4(y-2)^2 - 4 + k = 0 \\
 & (x-2)^2 + 4(y-2)^2 = 8 - k
 \end{aligned}$$

$$8 - k > 0$$

$$k < 8$$

$$k \in (-\infty; 8)$$

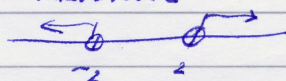
$$\begin{aligned}
 b) \quad & x^2 + y^2 + 2kx + 4y + 8 = 0 \\
 & (x+k)^2 - k^2 + (y+2)^2 - 4 + 8 = 0 \\
 & (x+k)^2 + (y+2)^2 = -4 + k^2
 \end{aligned}$$

$$k^2 - 4 > 0$$

$$(k-2)(k+2) > 0$$

JE TO ZVLÁŠTNÍ PŘÍPAD ELIPSY - KRUŽNICE

$$k \in (-\infty; -2) \cup (2; \infty)$$



30.

$$a) p: 2x + y - 6 = 0$$

$$c: 4x^2 + y^2 = 20$$

$$p \cap c: y = 6 - 2x \rightarrow 4x^2 + (6 - 2x)^2 = 20$$

$$4x^2 + 36 - 24x + 4x^2 = 20$$

$$8x^2 - 24x + 16 = 0 \quad | : 8$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x_1 = 1 \quad y_1 = 4$$

$$x_2 = 2 \quad y_2 = 2$$

$$P_1 [1; 4]$$

$$P_2 [2; 2]$$

$$b) p: x = 3$$

$$c: x^2 + 4y^2 = 16$$

$p \cap c:$

$$9 + 4y^2 = 16$$

$$4y^2 = 7$$

$$y^2 = \frac{7}{4}$$

$$P_1 [3; \frac{\sqrt{7}}{2}]$$

$$P_2 [3; -\frac{\sqrt{7}}{2}]$$

signa

$$y_1 = \frac{\sqrt{7}}{2}$$

$$y_2 = -\frac{\sqrt{7}}{2}$$

31.

$$a) c: 9x^2 + y^2 - 5 = 0$$

$$p: 3x - 2y = 0$$

$$t \parallel p: t: 3x - 2y + c = 0$$

$$c \cap t: 3x - 2y + c = 0$$

$$3x = 2y - c$$

$$(3x)^2 + y^2 - 5 = 0$$

$$(2y - c)^2 + y^2 - 5 = 0$$

$$4y^2 - 4cy + c^2 + y^2 - 5 = 0$$

$$5y^2 - 4cy + c^2 - 5 = 0$$

$$D = 0 \quad 16c^2 - 20(c^2 - 5) = 0$$

$$-4c^2 = -100$$

$$c^2 = 25$$

$$t_1: 3x - 2y + 5 = 0$$

$$t_2: 3x - 2y - 5 = 0$$

$$1) \quad x: x^2 + q^2 - x - 8q + 5 = 0 \quad 0 = x - 2q + 2 = 0$$

$$t \parallel f: x - 2q + 6 = 0$$

$$u \cap t: x = 2q - 6$$

$$(2q-6)^2 + q^2 - (2q-6) - 8q + 5 = 0$$

$$4q^2 - 4q + 6^2 + q^2 - 2q + 6 - 8q + 5 = 0$$

$$5q^2 - 10q - 4q + 6^2 + 6 + 5 = 0$$

$$5q^2 + q(-10-4) + 6^2 + 6 + 5 = 0$$

$$D = 0 \quad (-10-4)^2 - 4 \cdot 5(6^2 + 6 + 5) = 0$$

$$100 + 80q + 16q^2 - 20q^2 - 20q - 100 = 0$$

$$-4q^2 + 60q = 0$$

$$q(60-4q) = 0$$

$$q = 0$$

$$60-4q = 0$$

$$4q = 60$$

$$q = 15$$

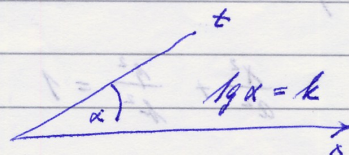
$$t_1: x - 2q = 0$$

$$t_2: x - 2q + 15 = 0$$

$$32. \quad x: x^2 + q^2 - x - q - 4 = 0$$

$$t: q = kx + q$$

$$q = x + q$$



$$\tan 45^\circ = k$$

$$1 = k$$

$$t \cap x: x^2 + (x+q)^2 - x - x - q - 4 = 0$$

$$x^2 + x^2 + 2xq + q^2 - 2x - q - 4 = 0$$

$$2x^2 + 2xq + q^2 - 2x - q - 4 = 0$$

$$2x^2 + x(2q-2) + q^2 - q - 4 = 0$$

$$D = 0 \quad (2q-2)^2 - 8(q^2 - q - 4) = 0$$

$$q = \pm 3$$

$$x - y - z = 0$$

$$u: x^2 + 2y^2 = 8$$

$$q_{1/2} = \frac{4 \pm \sqrt{16 + 48}}{6} = \frac{4 \pm 8}{6} \begin{cases} 2 \\ -\frac{2}{3} \end{cases}$$

$$P_2 \left[-\frac{8}{3}, -\frac{2}{3} \right]$$

$$|P_1 P_2| = \sqrt{\left(-\frac{8}{3} - 0\right)^2 + \left(-\frac{2}{3} - 2\right)^2} = \sqrt{\frac{64}{9} + \frac{64}{9}} = \frac{8}{3} \sqrt{2}$$

34.

a) org ma osach riadnic $\Rightarrow S(0;0)$

$$T_1[x_0, y_0]$$

$$T_2 \langle x_1, z_1 \rangle$$

$$t_2: \frac{x_1 \cdot x}{a^2} + \frac{\beta_1 \cdot \beta}{b^2} = 1$$

$$7x + 4y = 81$$

$$\frac{2}{81}x + \frac{4}{81}y = 1$$

$$\frac{f_1}{f^2} = \frac{4}{81}$$

$$f^2 = \frac{81}{4} \text{ g}$$

$$27x_0 = \frac{81}{2}x_1$$

$$189x_0 = 81x_1$$

$$\frac{27}{2}q_0 = \frac{81}{4}q_1$$

$$54q_0 = 81q_1$$

$$\begin{aligned} T_1 \in t_1 & \quad x_0 = 27 - 2q_0 \\ T_2 \in t_2 & \quad x_1 = \frac{-4q_1 + 81}{7} \end{aligned}$$

$$\begin{aligned} 27(27 - 2q_0) &= \frac{81}{7} \left(\frac{-4q_1 + 81}{7} \right) \\ 729 - 54q_0 &= \frac{81}{49}(-4q_1 + 81) \\ 54q_0 &= 81q_1 \end{aligned}$$

$$729 = \frac{81}{49}(-4q_1 + 81) + 81q_1$$

$$54q_0 = 81 \cdot 8$$

$$q_0 = 12$$

$$1 \cdot 81 - 35721 = -324q_1 + 6561 + 3268q_1$$

$$441 = -4q_1 + 81 + 48q_1$$

$$45q_1 = 360$$

$$q_1 = 8$$

$$x_0 = 27 - 2 \cdot 12 = 3$$

$$x_1 = \frac{-4 \cdot 8 + 81}{7} = 7$$

$$T_1 [3, 12] \quad T_2 [7, 8]$$

$$L: \frac{x^2}{81} + \frac{q^2}{162} = 1$$

$$a^2 = 27x_0 = 81$$

$$b^2 = \frac{27}{2}q_0 = 162$$

$$h) \quad t_1: x + q = 5 \quad t_2: x - 4q = 10$$

$$L: \frac{x^2}{a^2} + \frac{q^2}{b^2} = 1 \quad T_1 [x_0, q_0] \quad T_2 [x_1, q_1]$$

$$t_1: \frac{x \cdot x_0}{a^2} + \frac{q \cdot q_0}{b^2} = 1$$

$$t_2: \frac{x \cdot x_1}{a^2} + \frac{q \cdot q_1}{b^2} = 1$$

$$\frac{x}{5} + \frac{q}{5} = 1$$

$$\frac{x}{10} - \frac{2}{5}q = 1$$

$$\frac{x_0}{a^2} = \frac{1}{5}$$

$$\frac{q_0}{b^2} = \frac{1}{5}$$

$$\frac{x_1}{a^2} = \frac{1}{10}$$

$$\frac{q_1}{b^2} = -\frac{2}{5}$$

$$a^2 = 5x_0 \quad b^2 = 5q_0$$

$$a^2 = 10x_1$$

$$b^2 = -\frac{5}{2}q_1$$

$$5x_0 = 10x_1$$

$$5q_0 = -\frac{5}{2}q_1$$

$$x_0 = 2x_1$$

$$q_0 = -\frac{1}{2}q_1$$

$$T_1 \in t_1 \quad x_0 = 5 - q_0$$

$$T_2 \in t_2 \quad x_1 = 10 + 4q_1$$

$$5 - q_0 = 20 + 8q_1$$

$$q_0 = -\frac{1}{2}q_1$$

$$5 = 20 + \frac{15}{2}q_1$$

$$-15 = \frac{15}{2}q_1$$

$$-2 = q_1$$

$$q_0 = 1$$

$$x_0 = 5 - 1 = 4$$

$$x_1 = 10 + 4(-2) = 2$$

$$T_1 [4; 1]$$

$$T_2 [2; -2]$$

$$a^2 = 5x_0 = 20$$

$$b^2 = 5q_0 = 5$$

$$L: \frac{x^2}{20} + \frac{q^2}{5} = 1$$

35. *mag man machen* $\Rightarrow$ $SL(2; \mathbb{Q})$

$$w) M \in L \quad M [3; 4; 4]$$

$$L: \frac{x^2}{a^2} + \frac{q^2}{b^2} = 1$$

$$T [x_0, q_0]$$

$$M \in L: \frac{9}{a^2} + \frac{576}{b^2} = 1$$

$$\frac{x \cdot x_0}{a^2} + \frac{q \cdot q_0}{b^2} = 1$$

$$\frac{x}{\frac{25}{4}} + \frac{q}{5} = 1$$

$$\frac{x_0}{a^2} = \frac{4}{25}$$

$$\frac{q_0}{b^2} = \frac{1}{5}$$

$$a^2 = \frac{25}{4}x_0$$

$$b^2 = 5q_0$$

$$T \in L: 4x_0 + 5q_0 = 25$$

$$q_0 = \frac{25 - 4x_0}{5}$$

$$a^2 = \frac{25}{4}x_0$$

$$b^2 = 25 - 4x_0$$

$$\frac{9}{a^2} + \frac{5,76}{t^2} = 1$$

$$\frac{9}{\frac{25}{4}x_0} + \frac{5,76}{25-4x_0} = 1$$

$$9(25-4x_0) + \frac{25}{4} \cdot x_0 \cdot \frac{144}{25} = \frac{25}{4}x_0(25-4x_0)$$

$$225 - 36x_0 + 36x_0 = \frac{625}{4}x_0 - 25x_0^2 \quad | \cdot 4$$

$$4 \cdot 25x_0^2 - 625x_0 + 900 = 0$$

$$4x_0^2 - 25x_0 + 36 = 0$$

$$x_{0,1/2} = \frac{25 \pm 7}{8} = \begin{matrix} 4 \\ \frac{9}{4} \end{matrix}$$

$$q_0 = \frac{25-4x_0}{5} \rightarrow q_{0,1} = \frac{9}{5}$$

$$q_{0,2} = \frac{16}{5}$$

$$a^2 = \frac{25}{4} \cdot 4 = 25$$

$$a^2 = \frac{25}{4} \cdot \frac{9}{4} = \frac{225}{16}$$

$$t^2 = 25 - 16 = 9$$

$$t^2 = 25 - 4 \cdot \frac{9}{4} = 16$$

$$u_1: \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$u_2: \frac{16x^2}{225} + \frac{y^2}{16} = 1$$

$$h: M[4; -1] \quad t: x + 4y = 10$$

$$u: \frac{x^2}{a^2} + \frac{y^2}{t^2} = 1$$

$$M \in u: \frac{16}{a^2} + \frac{1}{t^2} = 1$$

$$t: T[x_0, q_0] \quad \frac{x \cdot x_0}{a^2} + \frac{y \cdot q_0}{t^2} = 1$$

$$\frac{x}{10} + \frac{4y}{10} = 1$$

$$\frac{x_0}{a^2} = \frac{1}{10}$$

$$T \in t: x_0 + 4q_0 = 10$$

$$x_0 = 10 - 4q_0$$

$$\frac{q_0}{t^2} = \frac{4}{10}$$

$$a^2 = 10x_0$$

$$t^2 = \frac{10}{4} q_0$$

$$\frac{16}{10(10-4q_0)} + \frac{1}{\frac{10}{4}q_0} = 1$$

$$\frac{8}{5(10-4q_0)} + \frac{2}{5q_0} = 1 \quad | \cdot 5q_0(10-4q_0) \quad 1 = \frac{8q_0}{10-4q_0} + \frac{2}{10-4q_0}$$

$$8q_0 + 2(10-4q_0) = 50q_0 - 20q_0^2$$

$$8q_0 + 20 - 8q_0 = 50q_0 - 20q_0^2$$

$$20q_0^2 - 50q_0 + 20 = 0 \quad | :10 \quad 2q_0^2 - 5q_0 + 2 = 0$$

$$2q_0^2 - 5q_0 + 2 = 0$$

$$q_{01/2} = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4} = \begin{matrix} 2 \\ \frac{1}{2} \end{matrix}$$

$$q_{01} = 2 \quad x_{01} = 10 - 4 \cdot 2 = 2 \quad T_1 [2; 2] \quad a^2 = 20 \quad k^2 = 5$$

$$q_{02} = \frac{1}{2} \quad x_{02} = 10 - 4 \cdot \frac{1}{2} = 8 \quad T_2 [8; \frac{1}{2}] \quad a^2 = 80 \quad k^2 = \frac{5}{4}$$

$$L_1: \frac{x^2}{20} + \frac{y^2}{5} = 1$$

$$L_2: \frac{x^2}{80} + \frac{4y^2}{5} = 1$$

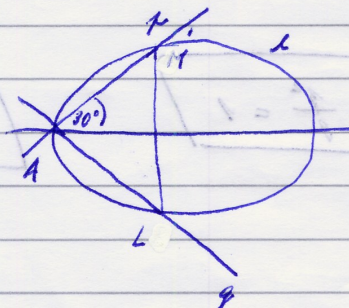
$$36. \quad L: x^2 + 9y^2 = 9$$

$$I. \quad S [0; 0] \quad A [-3; 0]$$

$$\frac{x^2}{9} + \frac{y^2}{1} = 1$$

$$a^2 = 9 \quad a = 3$$

$$k^2 = 1 \quad k = 1$$



$$p \cap L = \{A, C\} \quad p: y = kx + q \quad k = \tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$A \in p: 0 = -3 \cdot \frac{\sqrt{3}}{3} + q$$

$$q = \sqrt{3}$$

$$p: y = \frac{\sqrt{3}}{3}x + \sqrt{3}$$

$$L \cap p: x^2 + 9\left(\frac{\sqrt{3}}{3}x + \sqrt{3}\right)^2 = 9$$

$$x^2 + 9\left(\frac{3}{9}x^2 + 2x + 3\right) = 9$$

$$x^2 + 3x^2 + 18x + 27 = 9$$

$$4x^2 + 18x + 18 = 0 \quad | :2$$

$$2x^2 + 9x + 9 = 0$$

$$x_{1/2} = \frac{-9 \pm \sqrt{81 - 72}}{4} = \frac{-9 \pm 3}{4}$$

$$x_1 = -\frac{3}{2} \quad x_2 = -3 \rightarrow A$$

$$y_1 = \frac{\sqrt{3}}{2} \cdot \left(-\frac{3}{2}\right) + \sqrt{3} = -\frac{\sqrt{3}}{2} + \frac{2\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$M \left[-\frac{3}{2}; \frac{\sqrt{3}}{2} \right]$$

$$q: y = kx + q \quad k = \tan(-30^\circ) = -\frac{\sqrt{3}}{3}$$

$$y = -\frac{\sqrt{3}}{3}x + q$$

$$A \in q: 0 = +\sqrt{3} + q$$

$$q = -\sqrt{3}$$

$$y = -\frac{\sqrt{3}}{3}x - \sqrt{3}$$

$$L \cap q: x^2 + 9 \left(-\frac{\sqrt{3}}{3}x - \sqrt{3} \right)^2 = 9$$

$$x^2 + 9 \left(\frac{1}{3}x^2 + \dots \right)$$

NENÍ TČEBA

L je LM souměrní podle klamné osy $\Rightarrow L \left[-\frac{3}{2}; -\frac{\sqrt{3}}{2} \right]$

$$A[-3; 0]; L \left[-\frac{3}{2}; -\frac{\sqrt{3}}{2} \right]; M \left[-\frac{3}{2}; \frac{\sqrt{3}}{2} \right]$$

I. prochází mecholem $B[-3; 0]$; Δ^{BLM} je souměrný o ΔALM podle vedlejší osy

$$L' \left[\frac{3}{2}; -\frac{\sqrt{3}}{2} \right]$$

$$M' \left[\frac{3}{2}; \frac{\sqrt{3}}{2} \right]$$

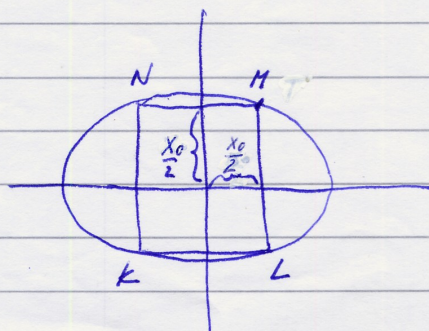
37.

$$L: x^2 + 3y^2 = 36$$

$\square KLMN$

$$M \left[\frac{x_0}{2}; \frac{x_0}{2} \right]$$

$$M \in L \quad \left(\frac{x_0}{2} \right)^2 + 3 \left(\frac{x_0}{2} \right)^2 = 36$$



$$4 \cdot \left(\frac{x_0}{2}\right)^2 = 36$$

$$\left(\frac{x_0}{2}\right)^2 = 9$$

$$\frac{x_0}{2} = 3$$

$$x_0 = 6$$

$$K[-3; -3]$$

$$M[3; 3]$$

$$L[3; -3]$$

$$N[-3; 3]$$

$$38. \text{ a: } \frac{x^2}{a^2} + \frac{y^2}{k^2} = 1$$

$$M[x_0; x_0]$$

$$M \in L: \frac{x_0}{a} = 1$$

$$\frac{x_0^2}{a^2} + \frac{x_0^2}{k^2} = 1$$

$$S = (2x_0)^2 = 4x_0^2$$

$$S = 4 \cdot \frac{a^2 k^2}{a^2 + k^2}$$

$$b = k^2 x_0^2 + a^2 x_0^2 = a^2 k^2$$

$$x_0^2 (a^2 + k^2) = a^2 k^2$$

$$x_0^2 = \frac{a^2 k^2}{a^2 + k^2}$$

