DERIVACE - ŘEŠENÍ

a) $y = x^4 - 2\sin x$

$y' = 4x^3 - 2\cos x$

b) $y = \frac{5}{x^3}$

$y = 5\left(x^3\right)^{-1} = 5\left(x^{-3}\right) \quad \Rightarrow \quad y' = 5 \cdot (-3) \cdot x^{-4} = -15x^{-4} = -\frac{15}{x^4}$

c) $y = 3^x + \sqrt{x^7}$

$y = 3^x + x^{\frac{7}{2}} \quad \Rightarrow \quad y' = 3^x \cdot \ln 3 + \frac{7}{2}x^{\frac{5}{2}} = 3^x \cdot \ln 3 + \frac{7}{2}x^2 \cdot \sqrt{x}$

d) $y = 3\ln x + 2\operatorname{cotg} x$

$y' = 3 \cdot \frac{1}{x} + 2 \cdot \frac{-1}{sin^2 x} = \frac{3}{x} - \frac{2}{sin^2 x}$

e) $y = x^2 \cdot \log_3 x$

$y' = 2x \cdot \log_3 x + x^2 \cdot \frac{1}{x ln 3} = 2x\log_3 x + \frac{x}{ln 3}$

f) $y = x^3 \cdot \sqrt[5]{x^2}$

$y = x^3 \cdot x^{\frac{2}{5}} = x^{3+\frac{2}{5}} = x^{\frac{17}{5}} \quad \Rightarrow \quad y' = \frac{17}{5}x^{\frac{12}{5}} = \frac{17}{5}x^2 \cdot \sqrt[5]{x^2}$

g) $y = \frac{x^2-9}{x+3}$

$y = \frac{(x-3)(x+3)}{x+3} = x - 3 \quad \Rightarrow \quad y' = 1$

h) $y = \frac{x^3+5}{x+3}$

$y' = \frac{(3x^2+0)(x+3)-(x^3+5)(1+0)}{(x+3)^2} = \frac{3x^3+9x^2-x^3-5}{(x+3)^2} = \frac{2x^3+9x^2-5}{(x+3)^2}$

i) $y = x^3 - 4x^2 + 6$

$y' = 3x^2 - 4 \cdot 2x = 3x^2 - 8x$

j) $y = \frac{1}{x^3-4x^2+6}$

$y = (x^3 - 4x^2 + 6)^{-1} \quad \Rightarrow \quad y' = -1\left(x^3 - 4x^2 + 6\right)^{-2} \cdot (3x^2 - 8x) = \frac{8x-3x^2}{(x^3-4x^2+6)^2}$

k) $y = \frac{\sqrt{x}\left(2x-x^4\right)}{\sqrt[3]{x}}$

$y = \frac{x^{\frac{1}{2}}(2x-x^4)}{x^{\frac{1}{3}}} = \frac{2x^{\frac{3}{2}}-x^{\frac{9}{2}}}{x^{\frac{1}{3}}} = 2x^{\frac{7}{6}} - x^{\frac{25}{6}} \quad \Rightarrow y' = 2 \cdot \frac{7}{6}x^{\frac{1}{6}} - \frac{25}{6}x^{\frac{19}{6}} = \frac{14}{6}\sqrt[6]{x} - \frac{25}{6}x^3\sqrt[6]{x} = \frac{1}{6}\sqrt[6]{x}(14 - 25x^3)$

l) $y = x^2 \cdot e^{\sin x}$

$y' = 2x \cdot e^{\sin x} + x^2 \cdot e^{\sin x} \cdot \cos x = xe^{\sin x}(2 + x\cos x)$

m) $y = \frac{\cos x^2}{x^2+1}$

$y' = \frac{-\sin x^2 \cdot 2x \cdot (x^2+1) - \cos x^2 \cdot (2x+0)}{(x^2+1)^2} = \frac{-2x(x^2+1)\sin x^2 - 2x\cos x^2}{(x^2+1)^2} = \frac{-2x[(x^2+1)\sin x^2 - \cos x^2]}{(x^2+1)^2}$

n) $y = \sqrt{x + \sqrt{x+1}}$

$y = \left(x + (x+1)^{\frac{1}{2}}\right)^{\frac{1}{2}} \quad \Rightarrow \quad y' = \frac{1}{2}\left(x + (x+1)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot (1 + \frac{1}{2}(x+1)^{-\frac{1}{2}} \cdot 1) = \frac{1+\frac{1}{2\sqrt{x+1}}}{2\sqrt{x+\sqrt{x+1}}}$

o) $y = e^{x \cdot \sin x}$

$y' = e^{x\cdot\sin x}\left(1 \cdot \sin x + x \cdot \cos x\right) = e^{x\cdot\sin x}\left(\sin x + x \cdot \cos x\right)$

p) $y = \sin\left(\frac{x^2-1}{2+x}\right)$

$y' = \cos\left(\frac{x^2-1}{2+x}\right) \cdot \frac{2x(2+x)-(x^2-1)\cdot 1}{(2+x)^2} = \cos\left(\frac{x^2-1}{2+x}\right) \cdot \frac{4x+2x^2-x^2+1}{(2+x)^2} = \frac{x^2+4x+1}{(2+x)^2} \cdot \cos\left(\frac{x^2-1}{2+x}\right)$

q) $y = (x+3)^3$

$y' = 3(x+3)^2 \cdot 1 = 3(x+3)^2$

r) $y = \frac{1}{\sin^2 2x + x^2 - 1}$

$y = \left(\sin^2 2x + x^2 - 1\right)^{-1} \quad \Rightarrow \quad y' = -1\left(\sin^2 2x + x^2 - 1\right)^{-2} \cdot (2\sin 2x \cdot 2 + 2x + 0) = \frac{-4\sin 2x - 2x}{(\sin^2 2x + x^2 - 1)^2}$

s) $y = \sin x^2$
$y' = \cos x^2 \cdot 2x = 2x \cos x^2$

t) $y = e^x \cdot \ln x$
$y' = e^x \cdot \ln x + e^x \cdot \frac{1}{x} = e^x \left(\ln x + \frac{1}{x}\right)$

u) $y = \cos 3x + \sin 3x$
$y' = -\sin 3x \cdot 3 + \cos 3x \cdot 3 = 3\left(\cos 3x - \sin 3x\right)$

v) $y = x^{e^{x^2}}$
$y' = e^{x^2} \cdot x^{(e^{x^2}-1)} \cdot e^{x^2} \cdot 2x$

w) $y = \frac{x^3+e^x}{\ln x}$ $y' = \frac{(3x^2+e^x)\ln x - (x^3+e^x)\frac{1}{x}}{\ln^2 x} = \frac{3x^2 \ln x + e^x \ln x - x^2 + \frac{1}{x}e^x}{\ln^2 x} = \frac{x^2(3\ln x - 1) + e^x(\ln x + \frac{1}{x})}{\ln^2 x}$

x) $y = \frac{\ln 3}{x}$
$y = \ln 3 \cdot x^{-1} \quad \Rightarrow \quad y' = \ln 3 \cdot (-1)x^{-2} = \frac{-\ln 3}{x^2}$

y) $y = x \cdot \ln x$
$y' = 1 \ln x + x\frac{1}{x} = \ln x + 1$

z) $y = \ln 2 \cdot 2$
$y' = 1$ Jedná se o derivaci konstanty.